

Rapid Generation of Optimal Asteroid Powered Descent Trajectories Via Convex Optimization

Robin Pinson (presenter)

NASA Marshall Space Flight Center

Ping Lu

Iowa State University

August 11, 2015

AAS/AIAA Astrodynamics Specialist Conference

AAS 15-616

Goal

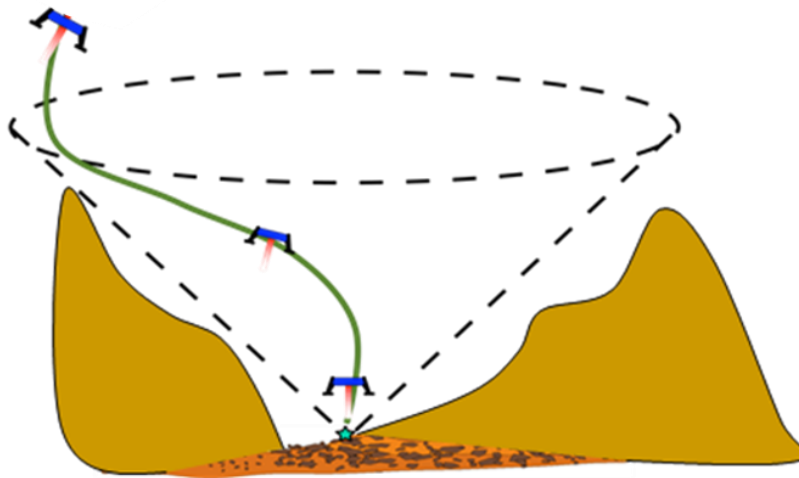
- Goal: Design an optimal powered descent trajectory on-board the spacecraft in order to softly land on an asteroid.
 - Algorithm would need to be autonomous, reliable, robust and efficient.
 - Designing on-board facilitates easy change of parameters.
- Convex optimization is efficient and reliable.
 - Guarantees global minimum in finite number of steps, if problem is feasible.
 - Especially true in subclasses, such as Second Order Cone Programming.
- ***Can convex optimization be used to design the asteroid powered descent trajectory?***

Original Problem Statement

- Asteroid powered descent fuel optimal problem is nonlinear and nonconvex.

$$\begin{aligned}
 & \min -m(t_f) \\
 \text{s.t. } & \dot{\vec{r}} = \vec{v}, \quad \dot{\vec{v}} = \frac{\vec{T}}{m} - 2\vec{\omega} \times \vec{v} - \dot{\vec{\omega}} \times \vec{r} - \vec{\omega} \times (\vec{\omega} \times \vec{r}) + \nabla U(\vec{r}), \quad \dot{m} = -\frac{1}{v_{ex}} \|\vec{T}\| \\
 & T_{min} \leq \|\vec{T}\| \leq T_{max}, \quad \|\vec{r}\| \cos\theta - \vec{r}^T \hat{n} \leq 0, \quad m \geq m_{dry} \\
 & \vec{r}(0) = \vec{r}_0, \quad \vec{v}(0) = \vec{v}_0, \quad m(0) = m_{wet}, \quad \vec{r}(t_f) = \vec{r}_f, \quad \vec{v}(t_f) = \vec{v}_f, \quad t_f \text{ given}
 \end{aligned}$$

- Highlighted terms are not permissible for a convex optimization problem.



Relaxation

- Original Problem

$$\min -m(t_f)$$

$$s.t. \dot{\vec{r}} = \vec{v}$$

$$\dot{\vec{v}} = \frac{\vec{T}}{m} - 2\vec{\omega} \times \vec{v} - \dot{\vec{\omega}} \times \vec{r} - \vec{\omega} \times (\vec{\omega} \times \vec{r}) + \nabla U(\vec{r})$$

$$\dot{m} = -\frac{1}{v_{ex}} \|\vec{T}\|$$

$$T_{min} \leq \|\vec{T}\| \leq T_{max}$$

$$\|\vec{r}\| \cos\theta - \vec{r}^T \hat{n} \leq 0$$

$$m \geq m_{dry}$$

$$\vec{r}(0) = \vec{r}_0, \vec{v}(0) = \vec{v}_0, m(0) = m_{wet}$$

$$\vec{r}(t_f) = \vec{r}_f, \vec{v}(t_f) = \vec{v}_f, t_f \text{ given}$$

- Relaxed Problem

$$\min -m(t_f)$$

$$s.t. \dot{\vec{r}} = \vec{v}$$

$$\dot{\vec{v}} = \frac{\vec{T}}{m} - 2\vec{\omega} \times \vec{v} - \dot{\vec{\omega}} \times \vec{r} - \vec{\omega} \times (\vec{\omega} \times \vec{r}) + \nabla U(\vec{r})$$

$$\dot{m} = -\frac{1}{v_{ex}} T_m$$

Slack Variable

$$\|\vec{T}\| \leq T_m$$

New Constraint

$$T_{min} \leq T_m \leq T_{max}$$

$$\|\vec{r}\| \cos\theta - \vec{r}^T \hat{n} \leq 0$$

$$m \geq m_{dry}$$

$$\vec{r}(0) = \vec{r}_0, \vec{v}(0) = \vec{v}_0, m(0) = m_{wet}$$

$$\vec{r}(t_f) = \vec{r}_f, \vec{v}(t_f) = \vec{v}_f, t_f \text{ given}$$

Proved the optimal solution of the relaxed problem is the optimal solution of the original.

Successive Solution Method

- Solve a series of convex optimization problems.
- Equations of motion can be arranged as:

$$\dot{\vec{x}}^{(k)} = A(r^{(k-1)}) \vec{x}^{(k)} + B\vec{u}^{(k)} + c(r^{(k-1)})$$

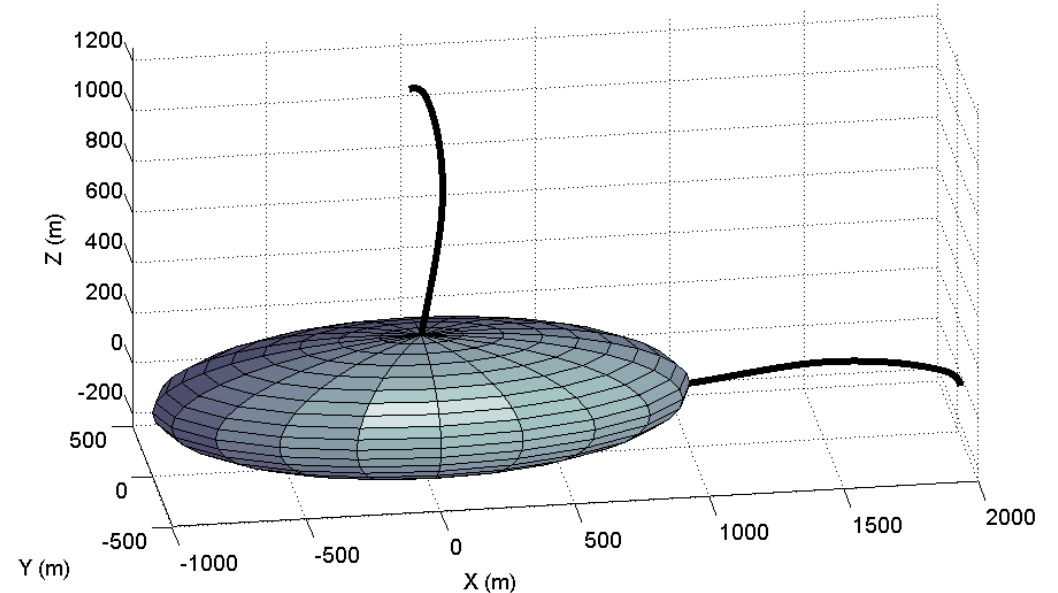
- A and c are evaluated using the previous solution (k-1).
- In the (k)th iteration, dynamics are linear and time varying.
- Iterations continue until two successive trajectories are within a set tolerance.

Scaling and Discretization

- Position, velocity, acceleration and time are scaled to keep the magnitudes near 1.
 - Position scaled by *smallest* semi-major axis
- Discretize the continuous equations to form equality constraints.
- **Now, the problem is in the form of a convex optimization problem.**
 - Linear equality constraints
 - Convex inequality constraints
- Used CVX, publically available Matlab based convex solver.
 - A 600 second trajectory, 300 nodes ~ 3300 variables per iteration, 4 iterations takes 3.5 minutes to complete.

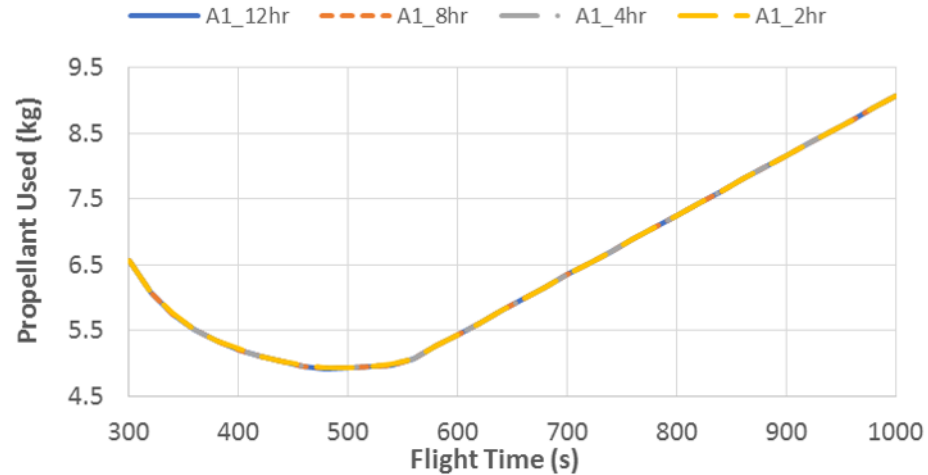
Asteroid Assumptions

- Two Trajectories
 - North Pole Trajectory (NP) lands on the z axis
 - Equatorial Trajectory (EQ) lands on the x axis
- Three Asteroid Sizes
 - A1: 1000 x 500 x 250 m
 - A2: 750 x 500 x 250 m
 - A3: 500 x 500 x 250 m
- Four Periods
 - 12 hr, 8 hr, 4 hr, 2 hr
 - Constant spin rate along z axis.

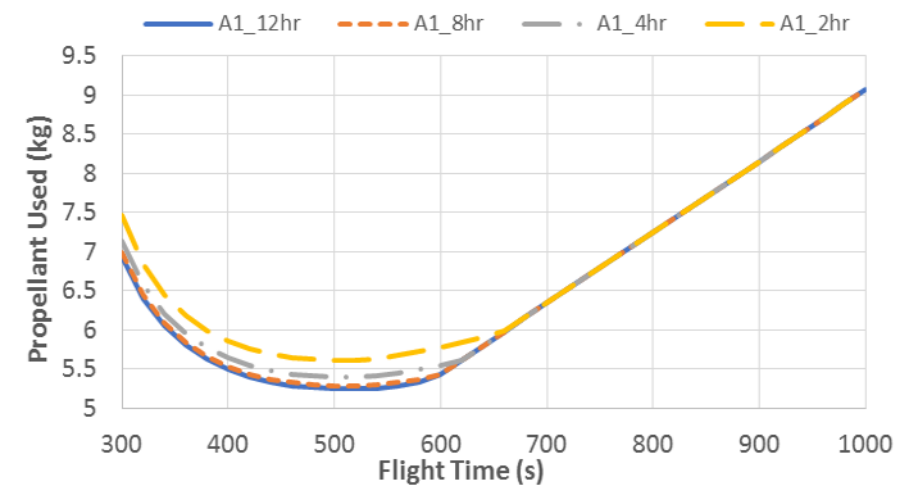


Propellant Usage Parameter Sweeps

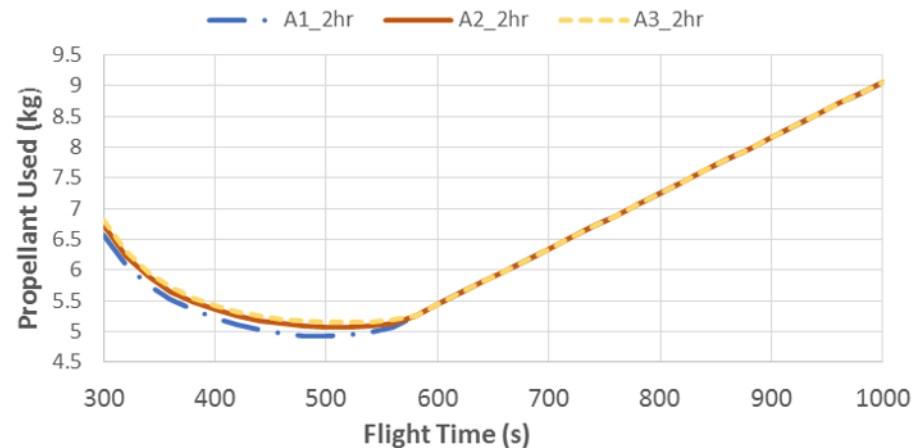
NP trajectories 1000 x 500 x 250 m



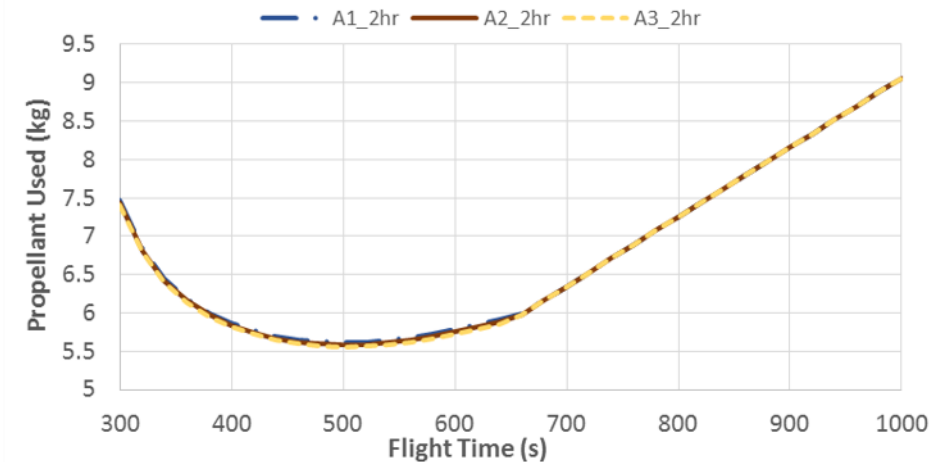
EQ trajectories 1000 x 500 x 250 m



NP trajectories 2 hr all asteroids

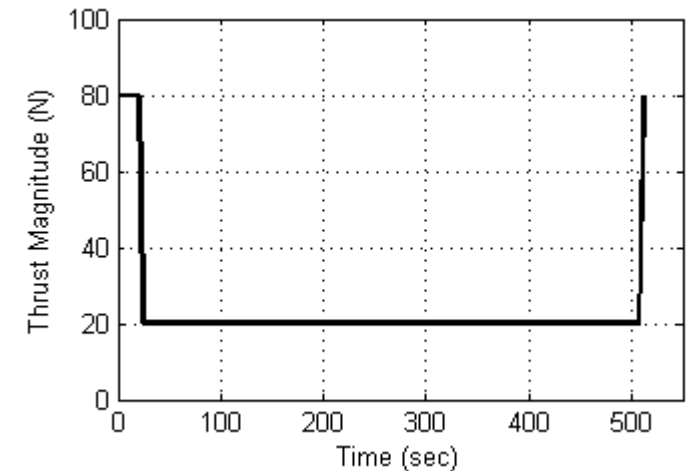
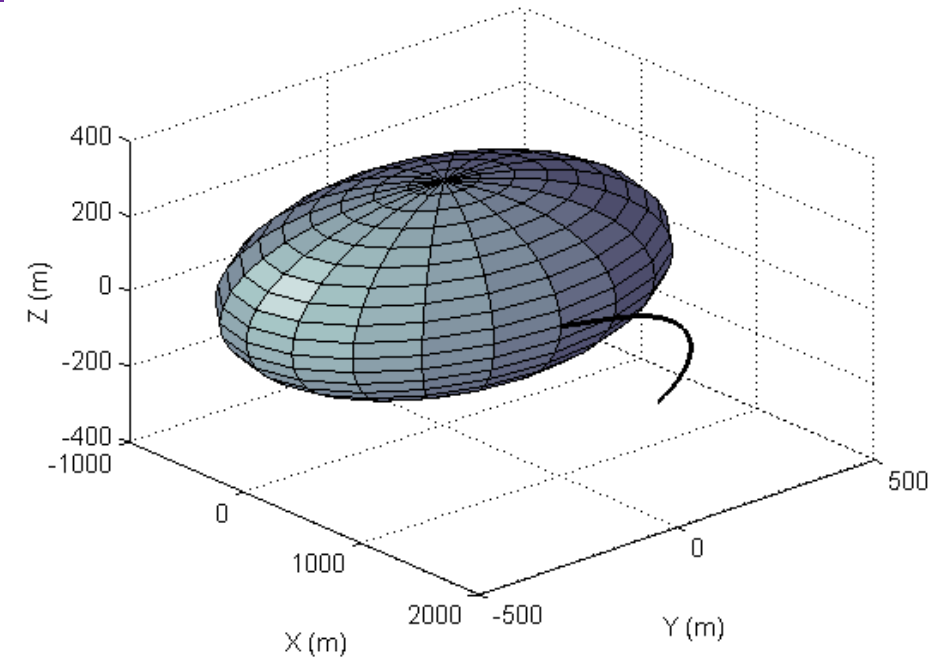
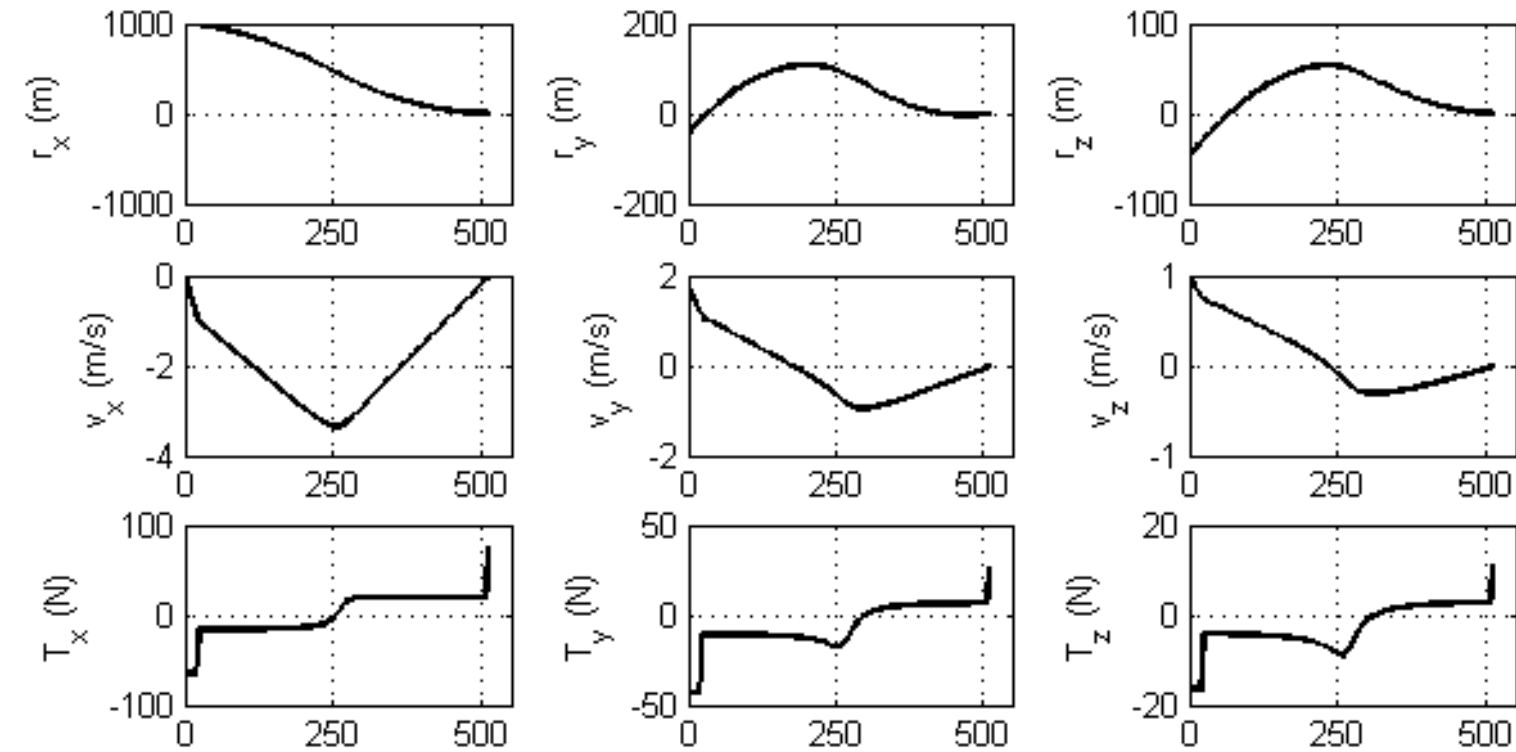


EQ trajectories 2 hr all asteroids



EQ Optimal Trajectory

- Minimum Propellant flight time 512 sec.
- 1000 x 500 x 250 m

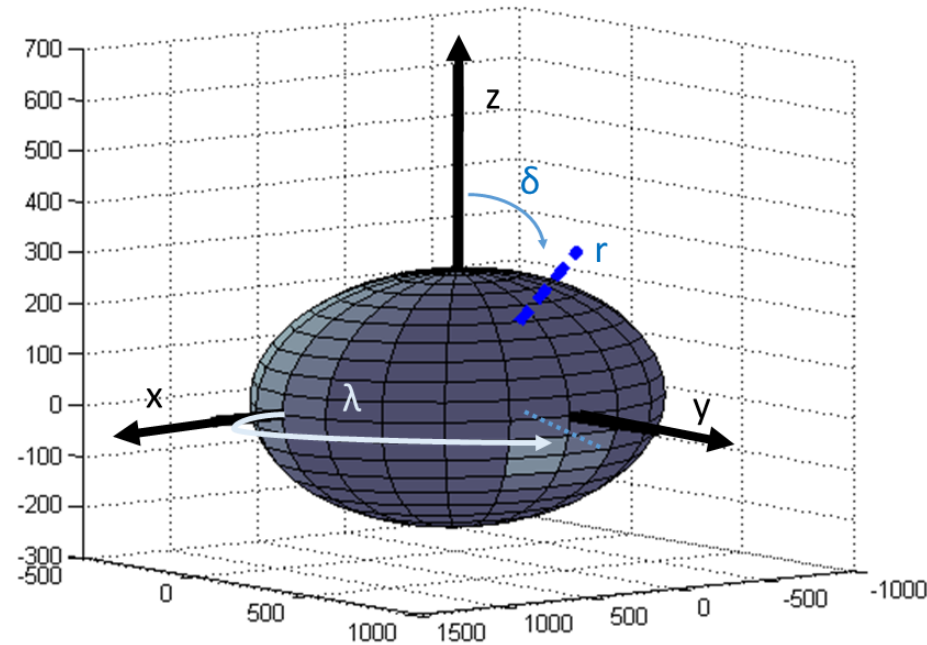


Conclusions

- Powered descent trajectory can be formulated as a convex optimization problem.
- Successive solution method is the key to handling nonlinear gravity model.
- Handles a wide range of parameters successfully.
- Viable algorithm for rapidly designing asteroid powered descent trajectories on-board the spacecraft.

-
- Back-up

Coordinates



- Use an asteroid centered fixed coordinate system to determine the spacecraft location, gravity and equations of motion.
 - x aligned with the largest axis, z aligned with the smallest axis, y completes the orthogonality.
 - Radius (r) is the distance from asteroid center to spacecraft.
 - λ spacecraft longitude
 - δ spacecraft latitude

Gravitational Acceleration

- Equations

$$\begin{aligned}\frac{\partial U}{\partial r_x} &= -\frac{\mu}{r^3} r_x + -\frac{\mu}{r^3} \left(G_1 r_x + G_2 \frac{r_x r_z}{(r_x^2 + r_y^2)^{0.5}} \right) + \frac{\mu}{r} G_3 r_y \\ \frac{\partial U}{\partial r_y} &= -\frac{\mu}{r^3} r_y + -\frac{\mu}{r^3} \left(G_1 r_y + G_2 \frac{r_y r_z}{(r_x^2 + r_y^2)^{0.5}} \right) - \frac{\mu}{r} G_3 r_x \\ \frac{\partial U}{\partial r_z} &= -\frac{\mu}{r^3} r_z + -\frac{\mu}{r^3} \left(G_1 r_z + G_2 \frac{r_z^2}{(r_x^2 + r_y^2)^{0.5}} \right) + \frac{\mu}{r} \frac{G_2}{(r_x^2 + r_y^2)^{0.5}}\end{aligned}$$

- Where

$$\begin{aligned}r &= (r_x^2 + r_y^2 + r_z^2)^{\frac{1}{2}} \\ \delta &= \arcsin \left(\frac{r_z}{r} \right) \\ \lambda &= \arctan \left(\frac{r_y}{r_x} \right) \\ G_1 &= \frac{1}{r^2} \{ 3C_{20} [1.5 \sin^2 (\delta) - 0.5] + 9C_{22} \cos^2 (\delta) \cos (2\lambda) \} \\ G_2 &= \frac{1}{r^2} \{ 1.5C_{20} \sin (2\delta) - 3C_{22} \sin (2\delta) \cos (2\lambda) \} \\ G_3 &= \frac{1}{r^2} \left\{ \frac{1}{(r_x^2 + r_y^2)} 6C_{22} \cos^2 (\delta) \sin (2\lambda) \right\}\end{aligned}$$

A, B, c

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ \omega^2 - \frac{\mu}{r^3} & 0 & 0 & 0 & 2\omega & 0 & 0 \\ 0 & \omega^2 - \frac{\mu}{r^3} & 0 & -2\omega & 0 & 0 & 0 \\ 0 & 0 & -\frac{\mu}{r^3} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^{(k-1)} \quad B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -\frac{1}{v_{ex}} \end{bmatrix}$$

$$c = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -\frac{\mu}{r^3} \left(G_1 r_x + G_2 \frac{r_x r_z}{\sqrt{(r_x^2 + r_y^2)}} \right) + \frac{\mu}{r} G_3 r_y \\ -\frac{\mu}{r^3} \left(G_1 r_y + G_2 \frac{r_y r_z}{\sqrt{(r_x^2 + r_y^2)}} \right) - \frac{\mu}{r} G_3 r_x \\ -\frac{\mu}{r^3} \left(G_1 r_z + G_2 \frac{r_z^2}{\sqrt{(r_x^2 + r_y^2)}} \right) + \frac{\mu}{r} \frac{1}{\sqrt{(x_k^2 + y_k^2)}} G_2 \\ 0 \end{bmatrix}^{(k-1)}$$

Spacecraft Parameters

- Thrust 80 N – 20 N
 - Isp 225 sec
- Mass 1400 kg
- Initial Conditions

	$r_{0x}(m)$	$r_{0y}(m)$	$r_{0z}(m)$	$v_{0x}(m/s)$	$v_{0y}(m/s)$	$v_{0z}(m/s)$
NP	-50	-50	1000+ γ	2	1	0
EQ	1000+ α	-50	-50	0	2	1